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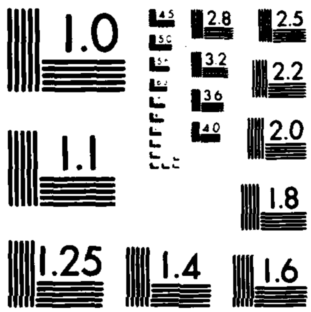

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ON OBTAINING PHYSICAL MEANING FROM THE PARAMETERS IN THE WEIBULL DISTRIBUTION

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The parameters (α,β) of the Weibull distribution do not lend themselves to physical interpretation. This report illustrates how the mean, median, mode, and other measures may be obtained from (α,β). Tables are given which display the mean, median, mode, and standard deviation as well as the cumulative probability of the distribution for one standard deviation above and below the mean for a number of different combinations of α and β.

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On Obtaining Physical Meaning From the Parameters in the Weibull Distribution

by

S. J. Bean and P. N. Somerville
University of Central Florida

1.0 Introduction

The Weibull distribution has been used in a number of applications. In climatological studies Somerville and Bean (1981, 1979) have found the Weibull distribution to fit visibility and windspeed well. The cumulative distribution function of the two parameter Weibull used in these studies is given by

$$F(x) = 1 - e^{-\alpha x^{\beta}}. \tag{1}$$

Other equivalent forms may be found in Johnson and Kotz (1970).

The parameters (α,β) do not lend themselves directly to interpretation such as mean (μ) and variance (σ^2) in the normal distribution. However, with a few calculations (α,β) can be used to obtain descriptive quantities for location and variability.

2.0 Measures of Location and Variability

The median, mode, percentiles, and interquartile range are directly available with only a few calculations. These quantities are found using the following formulas:

$$\text{Median} = [\ln(2)/\alpha]^{1/\beta} \tag{2}$$

$$\text{Mode} = [(\beta - 1)/\alpha\beta]^{1/\beta},\ \beta > 1 \tag{3}$$

$$x_p = [-\ln(1-p)/\alpha]^{1/\beta} \tag{4}$$

where x_p is the $100.p^{th}$ percentile. This expression for x_p is found by setting

$$p = 1 - \exp(-\alpha x_p^{\beta}) \qquad (5)$$

and solving for x_p. The median is found by letting $p = .5$. The mode may be found by taking the derivative of the frequency distribution and setting it equal zero and solving for x. The interquartile range may be used as a measure of variability. The interquartile range is the difference between the third and first quartiles or equivalently the 75th and 25th percentiles. That is

$$\text{Interquartile Range} = x_{.75} - x_{.25}. \qquad (6)$$

The percentiles may be used for obtaining prediction intervals. For example, 90% prediction intervals for a future observation of the variable x are given by $(x_{.05}, x_{.95})$ or $(x_{.01}, x_{.91})$. There are numerous other similar 90% prediction intervals. Each interval has the property that with approximate probability .9, the future observation will be contained within it.

The moments of the Weibull distribution are not as easily obtainable as the median, mode, and percentiles. The moments are defined in terms of gamma functions. The gamma function is defined as

$$\Gamma(p) = \int_0^\infty x^{p-1} e^{-x}\, dx, \quad p > 0 \qquad (7)$$

The mean (expected value) and variance of a Weibull variable x are given by

$$E[x] = \Gamma(1 + 1/\beta)/\alpha^{1/\beta} \qquad (8)$$

and

$$\mathrm{Var}[x] = [\Gamma(1 + 2/\beta) - (\Gamma(1 + 1/\beta))^2]/\alpha^{2/\beta}. \qquad (9)$$

The mean, median, mode, and standard deviation for various combinations of (α,β) are given in Tables 2.1, 2.2, 2.3 and 2.4 respectively.

For the normal distribution, the cumulative probability distribution of one standard deviation, σ, below the mean, μ, is always $F(\mu-\sigma) = P(X \le \mu-\sigma) = .1587$. Also the cumulative probability distribution of one standard deviation above the mean is always $F(\mu+\sigma) = P(X \le \mu+\sigma) = .8413$. Hence the probability of having any normal random variable within one standard deviation of its mean is always $F(\mu+\sigma) - F(\mu-\sigma) = .6814$. Unfortunately this is not true of the Weibull distribution. Since the mean and standard deviation can be obtained with a computer for the Weibull and since its cumulative distribution

$$F(x) = 1 - e^{-\alpha x^{\beta}}$$

is easily calculated, the comparable measures for the Weibull (i.e., $F(\mu-\sigma)$, $F(\mu+\sigma)$ and $F(\mu+\sigma) - F(\mu-\sigma)$) can be easily computed. Table 2.5 gives these values for a number of different combinations of α and β.

3.0 An Example

Visibility has been modeled for Fulda AAF, West Germany by Somerville and Bean (1981). At Fulda, for March 0900-1100 h local time

$$\alpha = .094, \ \beta = 1.247.$$

This yields the following

$$\begin{aligned}
\text{mode} &= ((.247)/(.094)(1.247))^{1/1.247} \\
&= 1.82 \text{ miles} \\
\text{median} &= x_{50} = (-\ln(.5)/.094)^{1/1.247} \\
&= 4.96 \text{ miles} \\
\text{mean} &= \Gamma(1 + 1/1.247)/(.094)^{1/1.247} \\
&= 6.2 \text{ miles}
\end{aligned}$$

At the same location for June 0900-1100 h

$$\alpha = .0027, \ \beta = 2.4966.$$

In this case we have the following estimates

$$\text{Mode} = 8.7 \text{ miles}$$

$$\text{Median} = 9.23 \text{ miles}$$

$$\text{Mean} = \Gamma(1.4)/(.0027)^{1/2.4966} = 9.48 \text{ miles}$$

TABLE 2.1
Mean for Weibull Distribution for Various Values of α and β

α \ β	.50	.75	1.00	1.50	2.00	2.50
.001	$2x10^6$	11096.4	1000.0	90.3	28.0	14.1
.010	$2x10^4$	552.6	100.0	19.4	8.9	5.6
.025	3200.0	162.9	40.0	10.6	5.6	3.9
.050	800.0	64.6	20.0	6.7	4.0	2.9
.100	200.0	25.7	10.0	4.2	2.8	2.2
.250	32.0	7.6	4.0	2.3	1.8	1.5
.500	8.0	3.0	2.0	1.4	1.3	1.2
.750	3.6	1.7	1.3	1.1	1.0	1.0
1.000	2.0	1.2	1.0	.9	.9	.9

TABLE 2.2
Median for Weibull Distribution for Various Values of α and β

α \ β	.50	.75	1.00	1.50	2.00	2.50
.001	$4.8x10^5$	6134.3	693.1	78.3	26.3	13.7
.010	4804.5	284.7	69.3	16.9	8.3	5.4
.025	768.7	83.9	27.7	9.2	5.3	3.8
.050	192.2	33.3	13.9	5.8	3.7	2.9
.100	48.0	13.2	6.9	3.6	2.6	2.2
.250	7.7	3.9	2.8	2.0	1.7	1.5
.500	1.9	1.5	1.4	1.2	1.2	1.1
.750	.9	.9	.9	.9	1.0	1.0
1.000	.5	.6	.7	.8	.8	.9

TABLE 2.3

Mode for Weibull Distribution for Various Values of α and β

	β		
α	1.50	2.00	2.50
.001	48.2	22.5	13.0
.010	10.5	7.2	5.2
.025	5.7	4.6	3.7
.050	3.6	3.3	2.8
.100	2.3	2.3	2.1
.250	1.3	1.5	1.5
.500	.9	1.1	1.2
.750	.7	.9	1.0
1.000	.6	.8	.9

(NOTE: There is no mode when $\beta \leq 1$.)

TABLE 2.4

Standard Deviation for Weibull Distribution for Various Values of α and β

	β					
α	.50	.75	1.00	1.50	2.00	2.50
.001	$4.5\text{x}10^6$	16107.7	1000.0	61.3	14.6	6.0
.010	44721.4	747.7	100.0	13.2	4.6	2.4
.025	7155.4	220.4	40.0	7.2	2.9	1.7
.050	1788.9	87.4	20.0	4.5	2.1	1.3
.100	447.2	34.7	10.0	2.8	1.5	1.0
.250	71.6	10.2	4.0	1.5	.9	.7
.500	17.9	4.1	2.0	1.0	.7	.5
.750	8.0	2.4	1.3	.7	.5	.4
1.000	4.5	1.6	1.0	.6	.5	.4

TABLE 2.5

Mean (μ), Standard Deviation (σ), Cumulative Probabilities ($F(\mu-\sigma)$ and $F(\mu+\sigma)$) and Probability of Being Within One Standard Deviation of the Mean ($F(\mu+\sigma) - F(\mu-\sigma)$) for the Weibull Distribution for Various Values of α and β

β	α	μ	σ	$F(\mu-\sigma)$	$F(\mu+\sigma)$	$F(\mu+\sigma) - F(\mu-\sigma)$
.500	.0010	2000000.00	4500000.00	.0000	.9219	.9219
	.0100	20000.00	44721.40	.0000	.9215	.9215
	.0250	3200.00	7155.40	.0000	.9215	.9215
	.0500	800.00	1788.90	.0000	.9215	.9215
	.1000	200.00	447.20	.0000	.9214	.9214
	.2500	32.00	71.60	.0000	.9215	.9215
	.5000	8.00	17.90	.0000	.9215	.9215
	.7500	3.60	8.00	.0000	.9223	.9223
	1.0000	2.00	4.50	.0000	.9219	.9219

β	α	μ	σ	$F(\mu-\sigma)$	$F(\mu+\sigma)$	$F(\mu+\sigma) - F(\mu-\sigma)$
.750	.0010	11096.40	16107.70	.0000	.8798	.8798
	.0100	552.60	747.70	.0000	.8853	.8853
	.0250	162.90	220.40	.0000	.8853	.8853
	.0500	64.60	87.40	.0000	.8852	.8852
	.1000	25.70	34.70	.0000	.8854	.8854
	.2500	7.60	10.20	.0000	.8854	.8854
	.5000	3.00	4.10	.0000	.8864	.8864
	.7500	1.70	2.40	.0000	.8848	.8848
	1.0000	1.20	1.60	.0000	.8852	.8852

β	α	μ	σ	F(μ-σ)	F(μ+σ)	F(μ+σ) - F(μ-σ)
1.000	.0010	1000.00	1000.00	.0000	.8647	.8647
	.0100	100.00	100.00	.0000	.8647	.8647
	.0250	40.00	40.00	.0000	.8647	.8647
	.0500	20.00	20.00	.0000	.8647	.8647
	.1000	10.00	10.00	.0000	.8647	.8647
	.2500	4.00	4.00	.0000	.8647	.8647
	.5000	2.00	2.00	.0000	.8647	.8647
	.7500	1.30	1.30	.0000	.8577	.8577
	1.0000	1.00	1.00	.0000	.8647	.8647

β	α	μ	σ	F(μ-σ)	F(μ+σ)	F(μ+σ) - F(μ-σ)
1	.0010	90.30	61.30	.1446	.8453	.7008
	.0100	19.40	13.20	.1431	.8445	.7015
	.0250	10.60	7.20	.1451	.8470	.7020
	.0500	6.70	4.50	.1505	.8465	.6960
	.1000	4.20	2.80	.1527	.8431	.6904
	.2500	2.30	1.50	.1638	.8431	.6793
	.5000	1.40	1.00	.1188	.8442	.7254
	.7500	1.10	.70	.1728	.8365	.6637
	1.0000	.90	.60	.1515	.8407	.6892

β	α	μ	σ	F(μ-σ)	F(μ+σ)	F(μ+σ) - F(μ-σ)
2.000	.0010	28.00	14.60	.1644	.8371	.6728
	.0100	8.90	4.60	.1688	.8384	.6696
	.0250	5.60	2.90	.1666	.8357	.6691
	.0500	4.00	2.10	.1651	.8444	.6793
	.1000	2.80	1.50	.1555	.8426	.6871
	.2500	1.80	.90	.1833	.8384	.6551
	.5000	1.30	.70	.1647	.8647	.6999
	.7500	1.00	.50	.1710	.8150	.6440
	1.0000	.90	.50	.1479	.8591	.7113

β	α	μ	σ	$F(\mu-\sigma)$	$F(\mu+\sigma)$	$F(\mu+\sigma) - F(\mu-\sigma)$
2.500	.0010	14.10	6.00	.1703	.8366	.6662
	.0100	5.60	2.40	.1674	.8364	.6690
	.0250	3.90	1.70	.1643	.8436	.6793
	.0500	2.90	1.30	.1495	.8359	.6865
	.1000	2.20	1.00	.1459	.8399	.6939
	.2500	1.50	.70	.1333	.8338	.7005
	.5000	1.20	.50	.1853	.8480	.6627
	.7500	1.00	.40	.1887	.8244	.6356
	1.0000	.90	.40	.1620	.8544	.6924

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